Cold scalar-tensor black holes coupled to a massless scalar field

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Abstract

New four-dimensional black hole solutions of Brans-Dicke equations with a negative constant ω, coupled to a massless scalar field, are presented. The temperature of these black holes is zero and the horizon area is infinite. An astrophysical application is also discussed.

Scalar-tensor theories of gravity are considered as the most natural generalizations of general relativity. From theoretical point of view one should mention that specific scalar-tensor theories arise naturally as a low energy limit of string theory. One of the most interesting aspects of scalar-tensor theories is the possible existence of black holes different form those in general relativity. Campanelli and Lousto [1] pointed out that, in Brans-Dicke theory with a negative coupling constant ω, there exist solutions possessing all black holes properties but with horizons of infinite area (type B black holes). Let us note that that Brans-Dicke theory with $\omega < 0$ is also worth to study for more than its intrinsic theoretical interest. For sufficiently negative ω, the theory passes all the experimental constrains as does for possitive ω [2].

Later on the work of Campanelli and Lousto was generalized in [3] where it was shown, in the framework of a general class of scalar-tensor theories, that nontrivial black hole solutions can exist for the coupling function $2\omega(\Phi) + 3 < 0$. These black holes solutions are divided into two classes: class $B1$, where horizons are attained by infalling particles in a finite proper time, and class $B2$, for which this proper time is infinite. The structure and stability of the scalar-tensor black holes were investigated in [4]. Charged scalar-tensor black holes were studied in [5] where it was shown that they exist for anomalous versions of the scalar-tensor theories with a negative kinetic term in the lagrangian. The temperature of these black holes is zero and the horizon area is (in most cases) infinite.

Thermodynamics of the black holes with infinite horizon area was considered in [6].

Stationary, axisymmetric black holes in Bans-Dicke theory with $-5/4 \leq \omega < -3/2$ were discussed in [7]. These black holes are characterize with zero Hawking temperature and finite horizon area.

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The aim of the present work is to present new type scalar-tensor black hole solutions of Brans-Dicke equations with $2\omega + 3 < 0$, coupled to a real massless scalar field.

The action for the scalar-tensor gravity in the presence of a real massless scalar filed σ is:

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(\Phi R - \frac{\omega}{\Phi}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - 2g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma\right) \tag{1}$$

The field equations are given by:

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R &= \frac{1}{\Phi}T_{\mu\nu} + \frac{\omega}{\Phi^2}\left(\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}g_{\mu\nu}\partial_\alpha\Phi\partial^\alpha\Phi\right) && (2)\\ &+ \frac{1}{\Phi}\left(\nabla_\mu\nabla_\nu\Phi - g_{\mu\nu}\nabla_\alpha\nabla^\alpha\Phi\right), && (3)\\ \nabla_\alpha\nabla^\alpha\Phi &= \frac{T}{3+2\omega}, && (4)\\ \nabla_\alpha\nabla^\alpha\sigma &= 0, && (5) \end{aligned}$$

where

$$T_{\mu\nu} = 2\partial_\mu\sigma\partial_\nu\sigma - g_{\mu\nu}\partial_\alpha\sigma\partial^\alpha\sigma \tag{6}$$

and $T = g^{\mu\nu}T_{\mu\nu}$.

We were able to find[1] the following static, spherically symmetric class of exact solutions of Eq.(2)-Eq.(6) for $2\omega + 3 < 0$:

$$\begin{aligned} ds^2 &= \left(\frac{1-\beta^2 f^b(r)}{1-\beta^2}\right)^2\left(-f^{a-b}(r)dt^2 + f^{-a-b}(r)dr^2 + f^{1-a-b}(r)r^2d\Omega^2\right), && (7)\\ \Phi^{-1}(r) &= \left(\frac{1-\beta^2 f^b(r)}{1-\beta^2}\right)^2 f^{-b}(r), && (8)\\ \sigma(r) &= \pm\beta \mid 3+2\omega \mid^{1/2} \frac{1-f^b(r)}{1-\beta^2 f^b(r)} && (9) \end{aligned}$$

where

$$f(r) = 1 - \frac{2\lambda}{r}. \tag{10}$$

The found class depends on three essential parameters λ, β ($\beta^2 < 1$) and a. The parameters b and a are related by

$$(2\omega+3)b^2 = 1 - a^2 \tag{11}$$

and therefore $a^2 \geq 1$. Here we shall consider the solutions with $\lambda > 0$.

[1]This class of exact solutions can be obtained by using solution generating techniques developed in [8].

Let us note that the usual vacuum Brans-Dicke solution (with $2\omega + 3 < 0$) is recovered for $\beta = 0$.

Black hole solutions with a metric

$$ds^2 = g_{00}(r)dt^2 + g_{11}(r)dr^2 + g_{22}(r)d\Omega^2 \tag{12}$$

are selected by the following criteria [3]:

1. There exists a Killing horizon: at some $r = r_*$, $g_{00}(r_*) = 0$;

2. The integral $\int \left(\frac{g_{11}}{|g_{00}|}\right)^{1/2} dr \to \infty$ as $r \to r_*$ (invisibility of the horizon for an observer at rest);

3. The Hawking temperaure T_H is finite;

4. The invariants $\mathcal{K}_1 = R^2$, $\mathcal{K}_2 = R_{\mu\nu}R^{\mu\nu}$, $\mathcal{K}_3 = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$ are finite at $r = r_*$.

It can be checked that all criteria are satisfied by the solutions with

$$1 < a, \;\; 0 < b, \;\; 2 - a \leq b < a. \tag{13}$$

Therefore, these solutions describe scalar-tensor black holes with a regular horizon at $r = r_* = 2\lambda$. These black holes are cold since $T_H = 0$ and have a horizon with an infinite area (i.e. type B black holes). It can be shown that the infalling particles reach the horizon for finite proper time when $0 < b < 1$ (type B1 black holes) and this time is infinite when $b \geq 1$ (type B2 black holes). It is worth also noting that the scalar field σ is regular on the horizon. Following the approach presented in [6] it can be shown that our black holes are characterized with zero entropy ($\mathcal{S} = 0$).

In this work we presented new type black hole solutions in Brans-Dicke theory with a negative coupling parameter ω : black holes coupled to a massless scalar field. These black hole solutions are interesting not only from a pure theoretical point of view. They, as well as the vacuum scalar-tensor black holes, could be of astrophysical relevance and some astrophysical applications were discussed in [1]. Here we would like to point out another astrophysical application. Gravitational lensing of the scalar-tensor black holes in the strong field regime could be quite different from that of the general relativistic black holes and could exhibit some new features. In fact, the gravitational lensing of objects (naked singularities) with a space-time metric similar to that of the scalar-tensor black holes was investigated by Virbhadra and Ellis in [9]. They found that the gravitational lensing of the strongly naked singularity is qualitatively different from that of a Schwarzschild black hole [10]. However, while the metric studied in [9] is characterized with a parameter $a < 1$ (in our notations, and $\nu < 1$ in the notations of [9]), the metrics of scalar-tensor black holes have $a > 1$. This could lead to qualitatively new effects. So, the strong field gravitational lensing may be used to distinguish between the scalar-tensor and general relativistic black holes. Of course, this question needs more careful investigation .

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